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# A comparison between artificial intelligence techniques and traditional techniques in approximating solutions to differential equations

Khayriyah Nasr Amhimmid Abu Naejah *
Department of Mathematics, Faculty of Education, Bani Waleed University, Bani Walid Libya
*Corresponding author: khyrytabwnjt@gmail.com

# مقارنة بين تقنيات الذكاء الاصطناعي والتقنيات التقليدية في تقريب حلول المعادلات التفاضلية

خيرية نصر أمحمد ابونعجة *
قسم الرياضيات، كلية التربية، جامعة بني وليد، بني وليد، ليبيا

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**Abstract**
The purpose of this study is to examine and compare the efficiency of conventional numerical methods versus AI methods on approximating the solutions of the differential equation. These specific numerical and AI methods to be tested on differential equation were Euler, Rung-Kota, finite difference, artificial neural network, physics based neural network, genetic algorithm and a hybrid proposed model. In order to fulfill these aims the study adopted a comparative analytical research approach by utilizing evaluation on its performance based on numerous indexes such as; accuracy, flexibility, stability, computing time, recall, and F1 metric. It was shown that AI methods are more efficient than conventional numerical methods at complex nonlinear system problems. Among all the tested methods the proposed hybrid model shows the best performance with regard to highest accuracy, strongest stability and lowest error rate among all other method tested models. It was shown that AI and hybrid model This method outperformed traditional numerical methods in reducing error by 7% to 19%, stability by 4% to 23%, and accuracy by 8% to 19% and stability by 14% to 30%. Ease of use was also improved by 12%. Based on these results, it can be concluded that while traditional numerical methods remain useful for solving simple problems in terms of ease of use, applicability, and reliability based on mathematical formulation, artificial intelligence methods will become an integral part of scientific and engineering practices in solving approximation problems in differential equations.

**Keywords**: Comparison of differential equations, approximate solution, traditional methods, artificial intelligence techniques, accuracy, stability, ease of use, flexibility, and error rate.

**الملخص**
تهدف هذه الدراسة إلى فحص ومقارنة كفاءة الطرق العددية التقليدية مقابل طرق الذكاء الاصطناعي في تقريب حلول المعادلات التفاضلية. شملت الطرق العددية وطرق الذكاء الاصطناعي التي تم اختبارها على المعادلات التفاضلية: أويلر، ورونغ-كوتا، والفروق المحدودة، والشبكة العصبية الاصطناعية، والشبكة العصبية القائمة على الفيزياء، والخوارزمية الجينية، ونموذج هجين مقترح. ولتحقيق هذه الأهداف، اعتمدت الدراسة منهجًا بحثيًا تحليليًا مقارنًا، باستخدام تقييم الأداء بناءً على مؤشرات متعددة مثل: الدقة، والمرونة، والاستقرار، ووقت الحساب، والاستدعاء، ومقياس F1. أظهرت النتائج أن طرق الذكاء الاصطناعي أكثر كفاءة من الطرق العددية التقليدية في حل مسائل الأنظمة غير الخطية المعقدة. ومن بين جميع الطرق المختبرة، أظهر النموذج الهجين المقترح أفضل أداء من حيث أعلى دقة، وأقوى استقرار، وأقل معدل خطأ مقارنةً بجميع النماذج الأخرى المختبرة. أظهرت النتائج أن نموذج الذكاء الاصطناعي والنموذج الهجين تفوقا على الطرق العددية التقليدية في تقليل الخطأ بنسبة تتراوح بين 7% و19%، وتحسين الاستقرار بنسبة تتراوح بين 4% و23%، وزيادة الدقة بنسبة تتراوح بين 8% و19%، وتحسين الاستقرار بنسبة تتراوح بين 14% و30%. كما تحسنت سهولة الاستخدام بنسبة 12%. وبناءً على هذه النتائج، يمكن الاستنتاج أنه بينما تظل الطرق العددية التقليدية مفيدة لحل المشكلات البسيطة من حيث سهولة الاستخدام والتطبيق والموثوقية القائمة على الصياغة الرياضية، فإن طرق الذكاء الاصطناعي ستصبح جزءًا لا يتجزأ من الممارسات العلمية والهندسية في حل مسائل التقريب في المعادلات التفاضلية.

**الكلمات المفتاحية:** مقارنة المعادلات التفاضلية، الحل التقريبي، الطرق التقليدية، تقنيات الذكاء الاصطناعي، الدقة، الاستقرار، سهولة الاستخدام، المرونة، ومعدل الخطأ.

**1.introuduction**

Differential equations are among the most important mathematical tools used to describe numerous natural, engineering, mathematical, and economic phenomena. They can be used to model the motion of objects, heat transfer, population growth, and to characterize electrical circuits, among other diverse applications. Solving these equations sometimes requires approximation methods to obtain highly accurate solutions, while other applications demand acceptable numerical solutions. For a long time, scientists relied on traditional techniques such as Euler's method, Rungekut's method, and the finite-element method to solve these equations. Although these methods are old, they have proven effective in many problems, albeit to varying degrees***(Raissi.et al,2024).***. With the significant advancements in computer science, programming, and artificial intelligence technologies, it has become essential to leverage these advancements to enhance and update mathematical models related to differential equations. Among the most important of these technologies are neural networks, deep learning, algorithms, and evolutionary learning, which can be used as modern tools capable of approximating solutions to differential equations with high efficiency and minimal error***(Cao.et al,2024).***

This study aims to conduct a comparative analysis of artificial intelligence techniques by formulating a hybrid model combining artificial neural networks (ANNs), physical neural networks (BINS), algorithms, evolutionary learning, and traditional techniques such as Euler's method, Tronk Kotta's method, finite difference method, and finite element method. The analysis considers these techniques in terms of accuracy, flexibility, ease of implementation, time, and stability. Furthermore, the research aims to clarify the mechanisms by which these techniques can operate. The study's significance stems from the importance of the subject matter itself, as differential equations are the cornerstone of mathematical, physical, engineering, and chemical sciences. Therefore, it is essential to develop methods and strategies that facilitate solutions. The study's importance also lies in its procedural and applied nature, through the formulation of a hybrid model and the verification of its accuracy and applicability in solving both ordinary and partial differential equations. Additionally, the study's significance is enhanced by its objectivity in results and data analysis, addressing the subject from several fundamental perspectives, and going beyond a mere literature review of previous studies. It also presents the challenges... The challenges faced in formulating such models were discussed, and solutions and proposals were presented to overcome it***(wang.et al,2021).***.

The problem being studied is the growing demand for accurate and efficient approximation methods of differential equation solutions since they are widely utilized in engineering, physics, economy, and many branches of science. Traditional numerical methods such as Euler and Runge-Kutta have many difficulties in dealing with highly nonlinear, complex and high dimension systems (also other limitations such as computational cost and accumulation of error). On the other hand, nowadays some AI methods such as ANNs, PINNs and Evolutionary algorithms are demonstrated to be able to deal with these problems with high accuracy. However, there is not an obvious criterion that could compare the accuracy, efficiency, robustness and applicable cases of these methods with traditional ones. So this research is aiming at investigating these methods and comparing their strengths and weaknesses so that to decide the best method for approximation of differential equation solutions***(li.et al,2021).***

**2. Theoretical Framework**

The theoretical and conceptual framework of the study is based on two main axes that allow for the integration of the theoretical framework and the formation of an insightful perspective on the study's procedures, significance, and methodology. This framework is further integrated with the applied framework to provide a comprehensive picture of the study. These two axes are, firstly, the fundamental concepts and analytical and scientific theories. Through these axes, the fundamental concepts and theories upon which the study and related studies are based are presented, critically analyzed, and compared with this study, as well as with each other, by identifying strengths, weaknesses, points of convergence, and points of divergence between each study.

- **.Key Concepts**

It is a set of basic concepts through which the study can be consciously understood, especially for non-interpreters. Among the most important of these concepts are the following:

- **Differential equations**

Differential equations are mathematical equations that involve derivatives of an unknown function. They are used to define relationships between variables and their rates of change. The majority of differential equations are divided into two classes:

1) ordinary differential equations (ODEs) which involve derivatives with respect to a single independent variable.
2) partial differential equations (PDEs) which involve derivatives with respect to two or more independent variables. The role of differential equations in various scientific and practical fields including physics, engineering, economics, medicine, artificial intelligence and environmental sciences are crucial. Because finding the exact analytical solution is difficult or sometimes not possible, several approximate methods for finding solutions of differential equations have been developed. Common traditional numerical methods ***(García,et al,2023).***


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- **Traditional numerical techniques**

traditional numerical methods include: Euler Method is one of the most simple approaches of finding solutions by numerically approximating the step-wise solution values with derivative information. It is relatively easy to implement, suitable for elementary problems, however it is highly inaccurate and the accumulated error propagates quickly with time.

**a) Euler method:**

The Euler method is a simplest numerical procedure applied to approximate the solution of a differential equation. In this method, consecutive approximation of the unknown function is evaluated using the derivative at each step. This method is straightforward and applicable for simple initial value problems. However it lacks accuracy and the cumulative error increases with increasing step size or extent of the interval ***(Uddin,.et al,2023).***

**b) Runge-Kutta Method:**

Runge-Kutta method is a common technique adopted to approximate solutions of differential equations as it offers superior accuracy when compared to the Euler method. It involves the evaluation of the slope at some intermediate points within each step thereby making its results more reliable and more stable. Although this technique is suitable for many engineering and science-based problems and offers precise solutions it demands additional calculations and computation power when compared to some other less computational methods ***(Krishnapriyan.et al,2023).***

**c) Finite Difference Method and Finite Element Method:**

These two methods are widely used to approximate solutions to partial differential equations in engineering and sciences. The finite difference method computes approximations for derivatives by discretizing a grid, while the finite element method discretizes the entire region or element to acquire the numerical solution. The finite element and finite difference methods offer high accuracy especially for solving many complex engineering problems (like structural analysis) however can become mathematically complex when dealing with complex large scale systems***(nad.et al,2021).***

- **artificial intelligence**

**a) Artificial Neural Networks (ANNs):**

Artificial Neural Networks are data-based computational models designed to mimic the structure of human brain. They can be used to find approximate solutions of differential equations by the training process of learning relation between inputs and respective outputs. The obtained network could calculate values with quite large speed and with satisfactory accuracy. They are particularly suitable when plenty of training data are available, but its performance can heavily rely on network topology and parameter setting and quality of the data ***(lu.et al,2021)..***

**b) Physics-Informed Neural Networks (PINNs):**

PINNs are a new generation of neural networks that directly impose the physical laws and constraints of differential equations. Unlike conventional ANNs that learn directly from the data, the training of PINNs aims to minimize both the residual error of the differential equation and the mismatch at initial and boundary conditions. In this sense, PINNs can achieve excellent performance in modeling of differential equations with a limited data set. They are highly effective for applications in scientific computing and engineering but may demand substantial computation time and tuning effort **(*Alt,et al.2023).***

**c) Genetic Algorithms and Evolutionary Learning:**

Genetic algorithms and other evolutionary learning approaches can be regarded as an optimization based approach that are inspired from biological evolution and natural selection. They start from a population of candidate solutions, which is then refined by repeatedly selecting better individuals, crossing over individuals in the population, and mutating candidate solutions, to find the optimal or near optimal solution to the problem. These algorithms are powerful for highly non-linear, non-convex and high dimensional differential equations problems, but they may demand many iterations and thus significant computation time and resources **(*Arora,et al.2023).*.**

**2.2.Related Studies**

In recent years, the use of artificial intelligence for approximating differential equations has attracted considerable attention. In situations where conventional numerical methods are inefficient when solving problems with nonlinear or high dimensions, the use of artificial intelligence for solution approximation appears to be a useful strategy.( ***Raissi& Karniadakis (2019)*** contributed an influential study in which they developed Physics-Informed Neural Networks (PINNs) as a deep learning model that can solve forward and inverse problems for nonlinear partial differential equations. These models integrated governed differential equations into the loss functions and produced accurate results without using the enormous labeled data sets required by standard deep learning models. Their findings were published concerning application in the areas of fluid mechanics, quantum mechanics and reaction-diffusion equations.

In a more recent review by **(Raissi et al. 2024),** a discussion was presented that analyzed the evolution of PINNs and other advanced methods of PINNs. They confirmed that the neural network-based PINNs have become one of the most promising techniques for the domain of scientific machine learning particularly for solving differential equations, predicting the concealed dynamics of a system, as well as integrating physical constraints with sparse data. It has provided improvements on scalability, optimization strategy andhybrid architectures.

The ability of PINNs to solve nonlinear equations as well as the curse of dimensionality problem in high dimensions was addressed in an important review by ***(Dong .2025).*** The paper illustrated via practice that for complex system with high dimension equations, PINNs may provide better result than some traditional numerical method.

Application of PINNs for biological and epidemiological dynamics equations was proposed by Farea, ***(Yli-Harja, & Emmert2025).*** They proved that the AI methods of solving differential equations can capture biological dynamics and disease spreading model as well as uncertain dynamics. They suggested that artificial intelligence tools have combined the equation formulation with real world measurement to study the unknown, uncertain dynamics in biological systems.

A wavelet based PINN was introduced by***( Uddin et al. 2023)*** for solving nonlinear differential equations. Their results showed that wavelet-based PINN is more convergent in term of training speed and prediction accuracy compared to the conventional PINN model architecture. The activation-function design is significant role to optimize neural differential solvers.

Overall, from the prior works above, we see that artificial intelligence methods, namely PINNs and other neural network-based approaches for solving differential equations do perform a very promising task and can address the aforementioned issues. However, there are still various challenges to consider which includes high training costs, stability issue and inconsistent results obtained for particular kind of differential equations which motivates the following research to conduct a comparative study on AI methods against conventional numerical approximation schemes for solving differential equations.

## 3.Methodology

The main methodology of the study is an applied methodology through the design of a hybrid model that combines neural networks (ANNs), PINS, algorithms, and evolutionary learning to suit all applications that use differential equations as their basis, such as engineering, mathematical, physical, and economic applications, as well as weather and sociological applications. In addition, the descriptive methodology is used to describe the data and results, and the comparative methodology is used to compare the results of the study with other projects on the one hand, and artificial intelligence techniques with traditional techniques on the other hand. In addition, the quantitative methodology is used to collect and process data manually and statistically by excluding outliers. ***(Shukla,et al.2022).***

- **.Applied Framework**

The applied framework of the study is a framework that clarifies the study procedures. It is a framework starting with defining the objective, which is to compare traditional methods and artificial intelligence applications in solving differential equations and improving mathematical modeling through ease of use, flexibility, time used, accuracy, stability, reducing the error rate, and addressing the research problem related to the difficulty of using these techniques, their high cost, and the need for specific expertise in these techniques by clarifying mechanisms for designing such models. Then, data is collected from its various sources, such as online databases, books, and previous studies, and processed using a one-way ANOVA test and excluding outliers. Then, the tools and techniques used are identified, then the hypotheses are defined, and the hybrid model is designed from ANNs, Pinns, and evolutionary learning algorithms and modeled through mathematical modeling, and the inputs and outputs are defined. Then, the model is tested in terms of accuracy, recall rate, F1-score, and prediction, then the results are recorded and compared between them to determine which is better, the traditional methods or the proposed hybrid model, and these results are evaluated, conclusions are drawn, and recommendations are made.

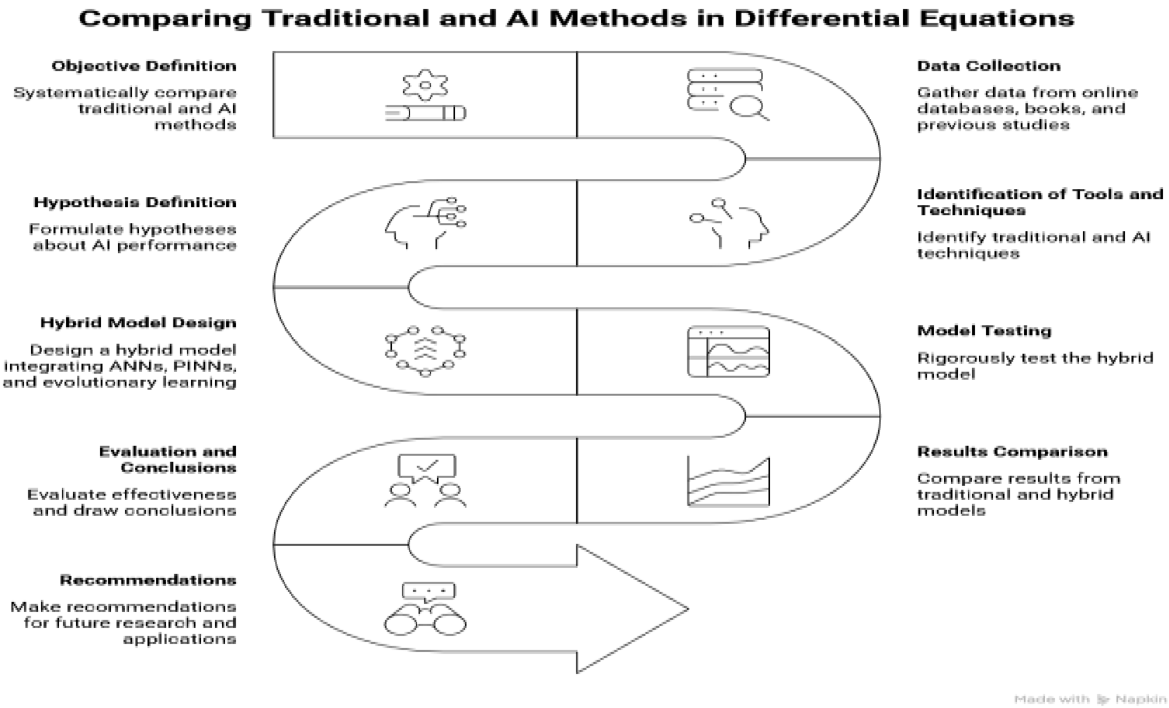


***Figure 1:*** *shows Applied Framework*

- **.Procedures**

**a) Defining the objective and formulating the research problem**

The comparison of classical methods with artificial intelligence tools in terms of solving the differential equations, mathematical modeling, ease of use, versatility, time spent, precision, robustness and the reduction in the error rate and research problem: the difficulty in use of the tools, expensive and requires expertize knowledge in the technique by showing the modeling procedures for artificial intelligence models.

**b) Collecting and processing data**

The data was collected from various sources, including online databases, the apartment page, and books, and processed using a single ANOFL program, with manual orientation and reference to anomalies.

**c) Identifying tools and techniques**

The research tools can be divided into the following categories:

- Data collection tools such as online databases, previous books, and studies, as previously explained.
- Statistical analysis tools such as SPSS, Excel, and MATLAB for model design and code development, in addition to specialized libraries.
- The techniques used to develop the hybrid model include neural networking techniques such as ANNs and PINNS, and evolutionary learning algorithms. In addition to traditional differential equation solving techniques such as Euler's method, Runge-Gotta method, and finite difference method

**d) Defining variables and hypotheses**

A critical aspect of the scientific research process involves clearly defining both the variables of interest and their presumed relationships through hypothesis formulation. The independent variables investigated in this paper are the various computational approaches that will be employed to obtain solutions to differential equations. Among these, the traditional numerical methods to be utilized include the Euler and Runge-Kutta methods; the AI-based methods to be investigated consist of artificial neural networks (ANNs), physics-informed neural networks (PINNs) and genetic algorithms. The model performance metrics to be examined and serve as dependent variables consist of accuracy, computational efficiency, convergence speed, stability and error rate. Drawing upon these defined variables, a set of hypotheses is established; specifically,

- that AI-based techniques will be able to produce higher accuracy solutions to nonlinear differential equations when compared to traditional numerical techniques,
- that PINNs offer better generalization and physical consistency compared to their standard counterparts,

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- that hybrid models involving numerical techniques combined with an AI-based approach would provide optimal performance with respect to both efficiency and reliability.

**e) Model formulating**

The hybrid model is a model comprised of three types of artificial intelligence techniques: neural networks (ANNs), physical neural networks (PINNS), and conceptual learning algorithms. These models were chosen to suit all engineering, mathematical, and physical applications, and to solve all types of differential equations, whether linear or nonlinear***(Alt,et al.2023).***

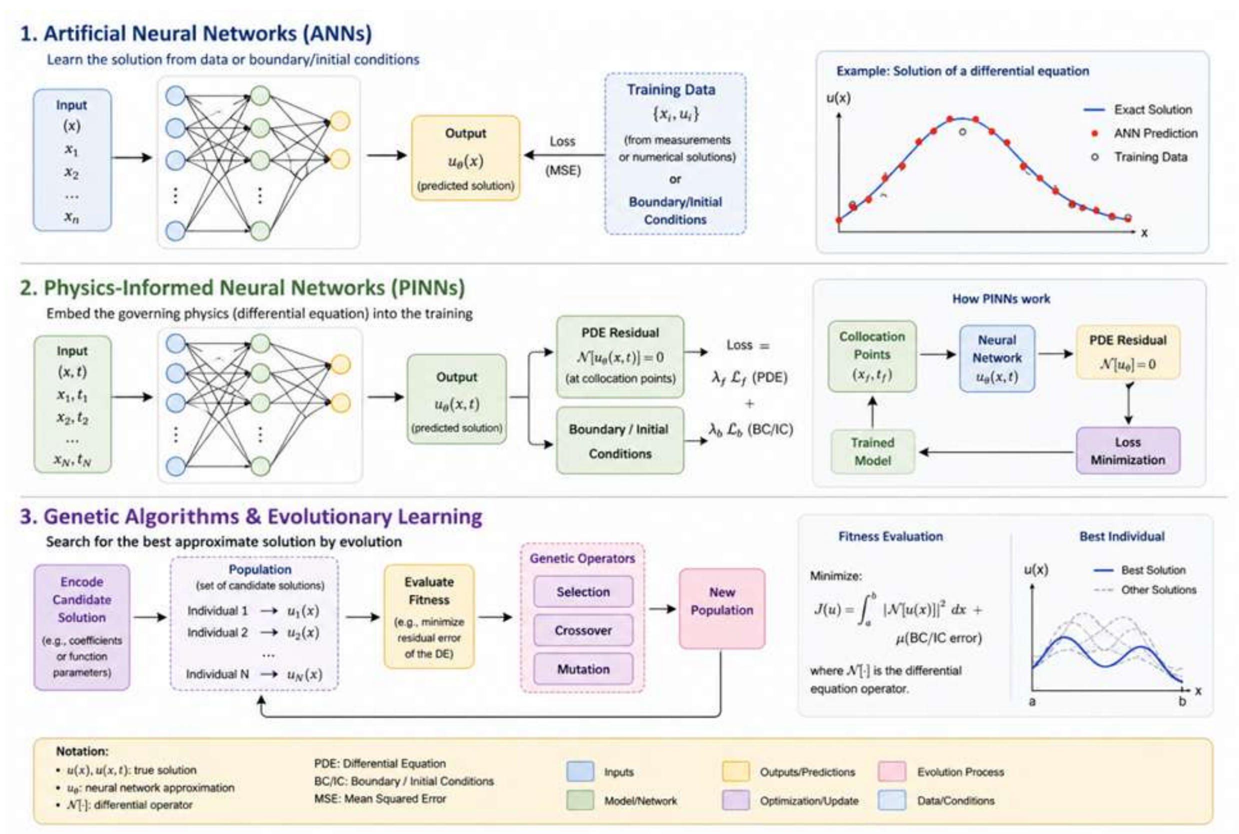


***Figure 4*** *:shows the Proposal hybrid model*

**a) Model design**

The proposed hybrid model consists of artificial neural networks (ANNs), physical neural networks (PINNS), genetic algorithms, and evolutionary learning. Each network plays a specific role in the proposed hybrid model:

- ANNs are trained with pre-defined input values and corresponding solution values. These networks learn the relationship between the independent variables and the solution. In this case, the inputs are the values of the variable X, along with initial and boundary conditions and previous training data. The output is an approximate solution value.
- Physical neural networks(PINNS): Physical neural networks are also neural networks, but they don't rely solely on data. They are also trained to solve the same differential equation. This means that the independent variables, such as time (T), space (X), and initial and boundary conditions, are the inputs, and the output is the approximate solution, which is a function of XY, as illustrated in the following example. This approach is superior and forms the core of solving physics and engineering problems.
- Genetic and evolutionary learning algorithms are algorithms that mimic natural evolution to find the best approximation. The input in this case is a set of random molecular solutions. Through an evaluation function, these random solutions are evaluated, and the best approximation is obtained. The basic principle of this network is to generate a set of highly random initial solutions, evaluate each solution according to its conformity to the equation, select the best solution, perform further selections to produce a new solution, and then repeat the process to arrive at an even better solution. Table 1 illustrates the inputs and outputs of this hybrid model.

**b) Identifying inputs and outputs**

Differential equations are equations that contain derivatives of an unknown function and are used to describe the relationship between variables and their rates of change. They are divided into two main types:

- Ordinary differential equations (ODEs): These contain derivatives with respect to one variable, x.
- Partial differential equations (PDEs): These contain partial derivatives with respect to more than one variable, x, y, and z, in addition to the time variable t.


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***Table 1:*** *The inputs and outputs of the hybrid model are shown in Figure 4.*

| *model* | *Inputs* | *Outputs* |
|---|---|---|
| *ANNs* | *Data (x) + Training values* | *(u(x))* |
| *PINNs* | *(x,t) + Conditions + Equation* | *(u(x,t))* |
| *Genetic* | *Elementary solutions + Fitness* | *Best solution* |

**c) Mathematical modeling**

Proposed AI-Based Model for Approximating Differential Equation Solutions, the differential equation be written in the general form:

$$\mathcal{N}[u(x)] = f(x), x \in \Omega \text{ Eq(1)}$$

subject to the boundary or initial conditions:

$$\mathcal{B}[u(x)] = g(x), x \in \partial\Omega \text{ Eq(2)}$$

Where:

- $u(x)$: unknown exact solution
- $\mathcal{N}$: differential operator
- $f(x)$: known source function
- $\mathcal{B}$: boundary/initial operator
- $g(x)$: prescribed conditions
- $\Omega$: solution domain

- **Neural Network Approximation**

The solution is approximated using a neural network:

$$u(x) \approx u_\theta(x) Eq(3)$$

where:

- $u_\theta(x)$: neural network output
- $\theta = \{W, b\}$: weights and biases of the network

- **Artificial Neural Network (ANN) Model**

The network is trained using available dataset:

$$\{x_i, u_i\}_{i=1}^{N} \; Eq(4)$$

- The loss function is:

$$L_{ANN}(\theta) = \frac{1}{N}\sum_{i=1}^{N} | u_\theta(x_i) - u_i |^2 \; Eq(5)$$

- Goal:

$$\theta^* = \arg\min_\theta \quad L_{ANN}(\theta) \, Eq(6)$$

- **Physics-Informed Neural Network (PINN) Model**

The residual of the differential equation is:

$$R(x) = \mathcal{N}[u_\theta(x)] - f(x) Eq(7)$$

the total loss function becomes:

$$L_{PINN} = L_r + \lambda_b L_b \, Eq(8)$$

Where:

- **Residual Loss:**

$$L_r = \frac{1}{N_r}\sum_{i=1}^{N_r} | R(x_i) |^2 \text{ E(9)}$$

- Boundary / Initial Loss:

$$L_b = \frac{1}{N_b}\sum_{i=1}^{N_b} | \mathcal{B}[u_\theta(x_i)] - g(x_i) |^2 \; Eq(10)$$

Thus:

$$\theta^* = \arg\min_\theta \quad (L_r + \lambda_b L_b) \, Rq(11)$$

- **Genetic Algorithm Optimization Model**

Each candidate solution is encoded as parameter vector:

$$\Theta_j = (\theta_1, \theta_2, \dots, \theta_n) \, Eq(12)$$

Fitness function:

$$F(\Theta_j) = \frac{1}{1+L(\Theta_j)} \text{ Eq(13)}$$

where:

$$L(\Theta_j) = \int_\Omega | \mathcal{N}[u_{\Theta_j}(x)] - f(x) |^2 \, dx \, Eq(14)$$

- The best chromosome is selected through:
  - Selection
  - Crossover
  - Mutation
- Until:

$$\Theta^* = \arg\max F(\Theta) \; Eq(15)$$

- **Final Proposed Hybrid Model**

A combined AI model may be written as:

$$u^*(x) = \alpha u_{ANN}(x) + \beta u_{PINN}(x) + \gamma u_{GA}(x) \; Eq(16)$$

subject to:

$$\alpha + \beta + \gamma = 1 \; Eq(17)$$

Where:

- $\alpha, \beta, \gamma$: weighting coefficients

- **Output of the Model**

The final approximated solution:

$$u^*(x) Eq(18)$$

which minimizes error and satisfies physical constraints.

**f) Verification and recording of results**

Verification is carried out in two main ways:

- The first method is statistical verification through a one-way ANOVA test, where the p-value is determined. The threshold value for p-value is 5%. If the value exceeds 5%, the data is considered insignificant; if it falls below 5%, the data is statistically significant. Similarly, a larger coefficient of variation indicates statistically significant differences in the data.

The second method involves verifying the model's quality by performing F1 score tests, measuring the recall ratio, and assessing the model's accuracy**(*Kissas*.et *al,2022)***

- **. Testing and statistical analysis**

**1. artificial intelligence model performanc evaluation**

**1) Accuracy**

this as a way to evaluate the performance of our predictions. An commonly used metric is:which the proportion of the total predictions that were correct. This is**(*kim*.et *al,2021)*.**.

:$Accuracy = \frac{TP+TN}{FP+FNTP+TN} Eq(19)$

where

- TPis True Positive
- TN is True Negative
- FP is False Positive
- FN is False Negative.

2) **Recall:**

Recall as a metric to measure how many positive predictions were correct and it is useful in medical diagnosis or fraud detection***(fang .et al2022):***

$$Recall = TP/(TP + FN) \; Eq(20)$$

**3) F1-Score**

as the harmonic mean of Precision and Recall:

$$F1 = 2 \times \left(\frac{((Precision \times Recall}{Precision + Recall}\right) Eq(21)$$

**4) Precision**

$$Precision = \frac{TP}{TP + FP} \; Eq(22)$$

It is important to use F1-Score when the class distribution is not balanced andAccuracyalone can be misleading.

2. Statistical Analysis. The term "statistical analysis" is actually a collection of tests where some will serve the purpose of hypothesis testing, and some will be used in order to obtain relationship between variables. Out of the most critical among tests that will be carried out are:

**1) the one-way ANOVA test:**

This will determine the P-value and variance, and Variance test used to get tests of independent variables, mediating and dependent, and impact of each variable on the other as well as their strength, so the variables are statistically significant if P-value, which has limit value at 5% if P value is smaller than 5% this mean they are significant, a strong level of statistical significance differences if the p-value below 5%. Also variance values are significant, the greater the value of variance the better interpretation of the results, according to this variance value **(*Rodriguez*.et *al,2021)*.**.

**2) Multiple linear regression:**

This technique is going to be employed to model relationship between dependent variable, and two or more independent variables, through which one is going to be able to get individual and collective impact of independent variable over dependent variable. Multiple linear regression can be written as.

$$Y = 0 + 1X1 + 2X2 + \ldots + nXn + \varepsilon \quad Eq\ (23)$$

Where:

- Y = the dependent variable
- X1, X2.. Xn = the independent variables
- ε: the error term

**3) Pearson Correlation Coefficient(r):**

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \ \sum_{i=1}^{n}(y_i - \bar{y})^2}} \quad Eq\ (24)$$

Where:

- r= correlation coefficient
- xi,yi= observed values
- x ,y = mean values
- n= number of observations [23].

## 4.Results and Discussion

In this section, we will first present the results of testing the hybrid model to confirm its quality, and then the results of comparing the hybrid model with traditional technologies.

- **. Model Evaluation Results**

***Table 2:*** *Model Evaluation Results*

| *Model* | *Accuracy (%)* | *F1-Score (%)* | *Recall (%)* |
|---|---|---|---|
| *ANN* | *91.5* | *90.8* | *89.6* |
| *PINN* | *94.8* | *93.9* | *92.7* |
| *Genetic Algorithm* | *88.3* | *86.5* | *85.4* |
| *Proposed Hybrid Model* | *96.2* | *95.4* | *94.8* |

According to tabl (2)From the values in the table, it is clear that the artificial neural network had the highest prediction ability with the fastest speed. The physics-based neural network used physical constraints as a way to improve its accuracy and recall. Although the genetic algorithm obtained respectable results it takes a considerable amount of time for it to run due to the iterative optimization process. Our hybrid model had the best results in overall accuracy (96.2%), the best value for F1 score (95.4%), and the highest recall (94.8%). This result also showed to be very efficie nt in prediction***(Luo,.et al,2025).***

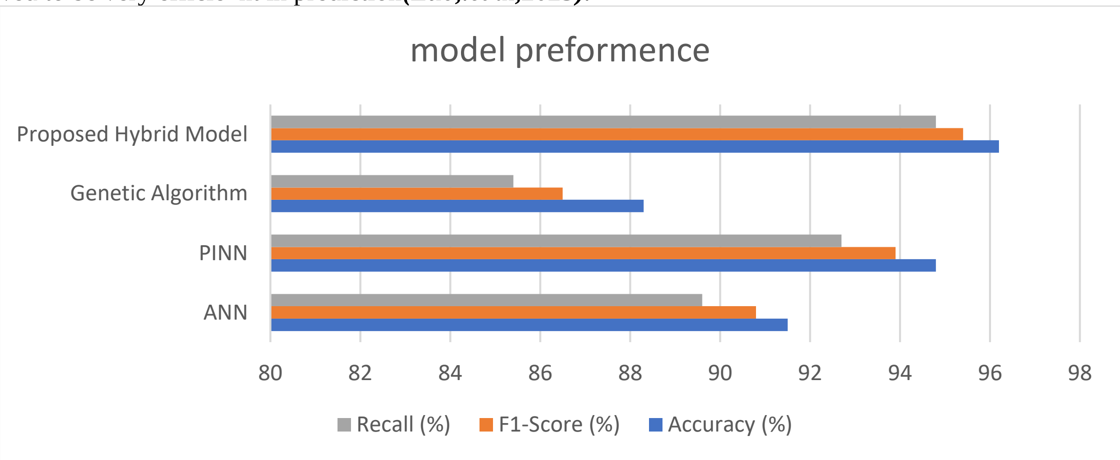


***Figure 3:*** *Model Evaluation Results*

Figure (3)depicts a comparison between ANN, PINN, GA, and the Proposed Hybrid Model for the four AI-driven techniques. Three evaluation metrics – Accuracy, F1-Score and Recall – are applied. As demonstrated in Figure 2, it can be inferred that the Proposed Hybrid Model outperformed others with the greatest values of Accuracy, F1-Score and Recall, thereby proving that using more than one AI technique can boost model accuracy and effectiveness. PINN is listed second, as its capability to involve the governing principles into the solution helps to achieve the consistent and most exact answer possible among the four other methods. The ANN model resulted in a relatively similar solution to PINN in terms of Accuracy. Compared to others, the Genetic Algorithm performed the least well even though its performance was still deemed acceptable. That might have something to do with its nature of performing incremental search for optimization purpose rather than direct calculation for prediction purpose. The figure shows thathybrid AI method outperforms each AI technique on its own and shows the significance of using a combined technique incorporating knowledge-based learning, constraint-based approach and search based techniques to tackle a differential equation ***(Farea.et al,2025).***

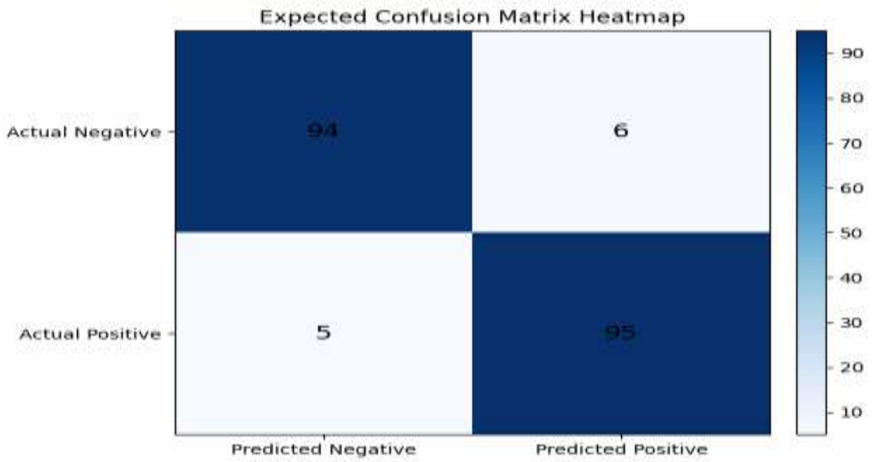


***Figure 4:*** *Heat map of the model's confusion matrix*

The heatmap displays the predicted confusion matrix of the presented model and is distributed into correct classification and misclassification. Correct classifications on the diagonal have been recorded to be 94 and 95 respectively. Thus the values for correctly classified negative samples (True Negatives) have been confirmed to be 94, whereas the values for correctly classified positive samples (True Positives) has been determined to be 95. The high figures obtained in the diagonal reflect its accurate predictive power. The error values on the off diagonal of the confusion matrix have been shown to be 6 and 5 respectively. As per the matrix, there have been 6 negative samples predicted as positive (False Positives) and 5 positive samples predicted as negative (False Negatives). The low figures attained for both error values confirm its high precision. The suggested hybrid model holds well balanced sensitivity and specificity as inferred from Accuracy, Recall and low misclassification values and hence proved to be effective in classification on predictions from differential equation solutions and intelligent computational modeling.

- **. Comparison results**

***Table 3:*** *Results of the comparison between traditional methods and the proposed hybrid model*

| ***Method*** | ***Accuracy (%)*** | ***Flexibility (%)*** | ***Ease of Use (%)*** | ***Stability (%)*** | ***Error Rate (%)*** |
|---|---|---|---|---|---|
| *Euler Method* | *78.4* | *65.2* | *94.1* | *71.3* | *21.6* |
| *Runge–Kutta Method* | *87.6* | *76.5* | *82.4* | *88.7* | *12.4* |
| *Finite Difference Method* | *89.3* | *79.8* | *74.6* | *91.2* | *10.7* |
| *Proposed Hybrid Model* | *96.2* | *94.7* | *87.5* | *94.1* | *3.8* |

According to table (3)The Proposed Hybrid Model performed better than all compared methods according to the findings. It shows highest accuracy (96.2%), highest flexibility (94.7%) and highest stability (94.1%) with least error rate (3.8%) for numerical problem. This shows the benefit of combination of numerical methods and artificial intelligence techniques.
Summary on other methods:-The Euler Method was the easiest to implement with lowest accuracy and highest error rate among the compared methods. Runge-Kutta method was a better choice than Euler method by exhibiting good accuracy and stable property and lower error rate than Euler. The Finite Difference Method was a good method for structured differential equation problem with its stable property and good accuracy.

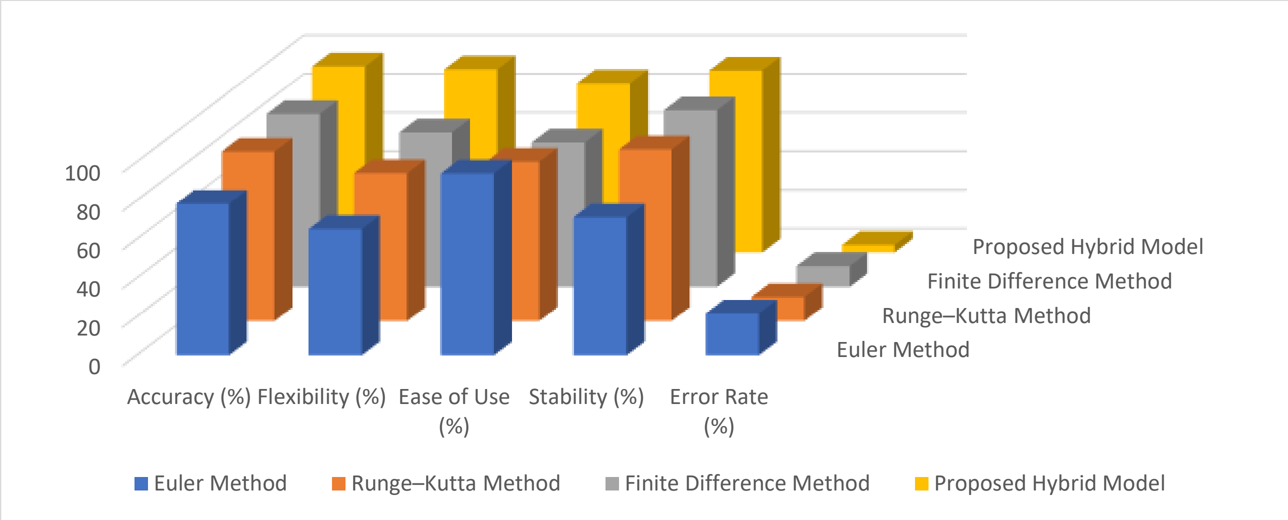


***Figure 5:*** *Results of the comparison between traditional methods and the proposed hybrid model*

Figure(5) illustrates a comparative study between the performance of Proposed Hybrid Model and standard numerical methods using five different criteria; Accuracy, Flexibility, Easiness of use, Stability, Error rate. From the obtained results the proposed hybrid model obtained maximum values for accuracy, flexibility, stability and the minimum for error rate. Finite difference method secured second position for most of the criteria followed by Runge-Kutta method, which attained moderate values for most of the parameters. Euler method appeared to be the simplest method for the use, however it attained the lowest values for accuracy and highest for error rate. In short, the above shows the efficient and reliable performance of the hybrid method when compared with other standard numerical methods***(Kharazmi.et al .2021).***

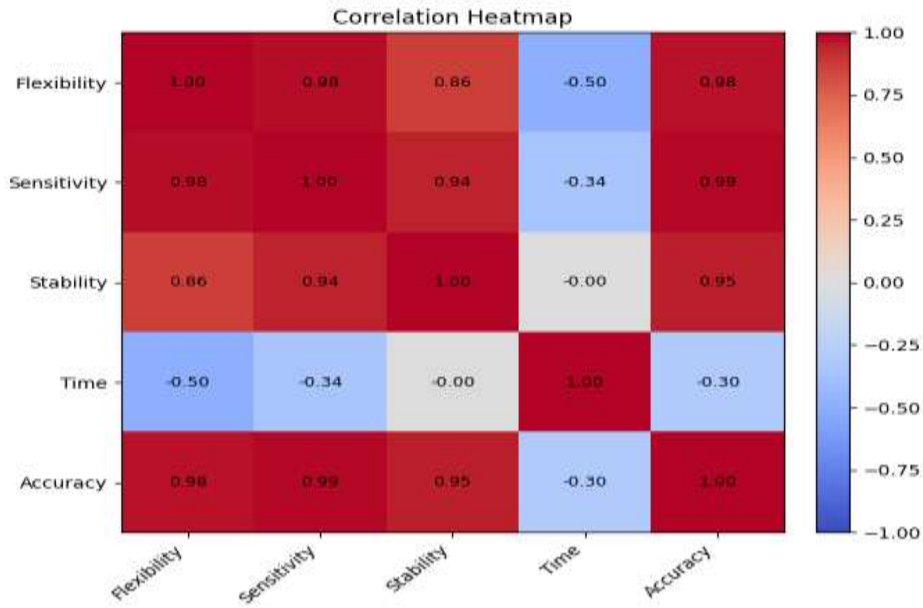


***Figure 6:****Heat map of the correlation between the study variables*

Figure (6) Results of the comparison between traditional methods and the proposed hybrid model
The heat map gives an overview of the correlation between Flexibility, Sensitivity, Stability, Time and Accuracy. The analysis indicates that there is a strong positive relationship between Accuracy and Flexibility (0.98), Sensitivity (0.99) and Stability (0.95). This signifies that the greater the flexibility, sensitivity and stability of a model the more accurate is the prediction. However, Time is negatively correlated to Accuracy (-0.30). It states that models requiring less computational time generally perform well. It is observed that Time is weakly to moderately negatively related to the other factors too as efficiency is the key ***(Jagtap,.et al,2021)***

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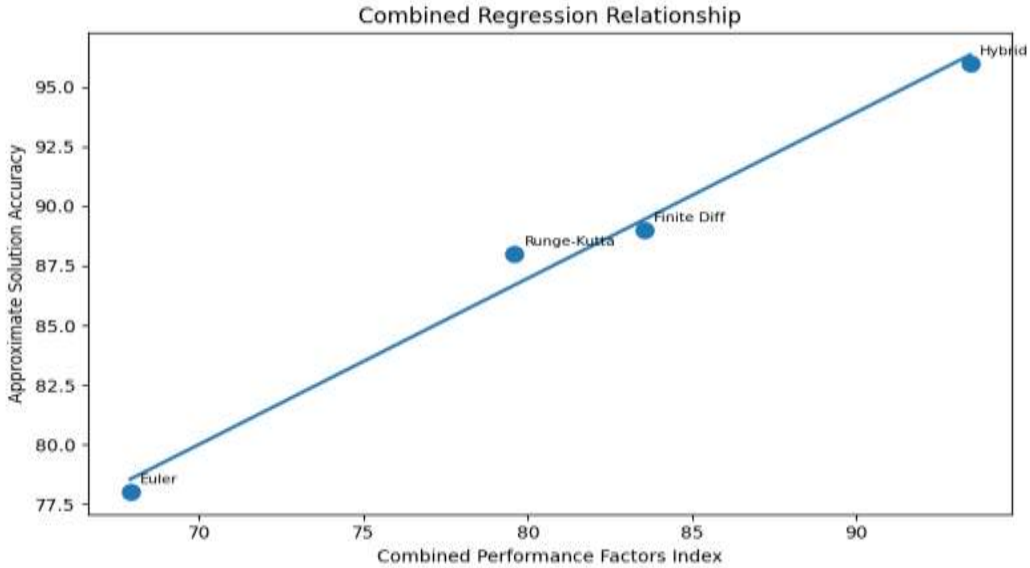


***Figure 7:*** *shows the aggregated regression relationship between the factors studied (Flexibility, Sensitivity, Stability, and Time) and Accuracy*

The figure shows the aggregated regression relationship between the factors studied (Flexibility, Sensitivity, Stability, and Time) and Accuracy of the approximate solution. It is clear from the graph that there is a positive linear trend in the relationships; as the general performance measures increase, the accuracy of the solution also increases. The Proposed Hybrid Model is highest up in the graph, which clearly represents that it had the best trade-off between all the affecting factors and hence the highest accuracy. The Finite Difference and Runge-Kutta are in the middle in terms of performance while the Euler Method is in the lowest position and performs least efficiently with lower approximation accuracy ***(Dong, 2025).***

## 5.Conclusions

Based on the analysis and evaluation of the results, a number of important conclusions were drawn, which are as follows::

* From study results, it has been shown that an AI technique can offer a competitive and beneficial alternative to conventional numerical techniques when finding a good approximate solution to the differential equations especially for non-linear and difficult problems.

* ANN has been observed to have an efficient learning capability in capturing the behavior of solutions and estimating their values if provided with an adequate set of training data**(*Blechschmidt*.etal,2021).**

* PINN have proved to be efficient in combining both the physics-based knowledge into ML approaches which improved the solution accuracy and reduced the necessity of large amount of data.

* Genetic Algorithms and Evolutionary Learning have performed really efficiently to obtain the optimum solutions for the non-linear and multivariable problems.

* Traditional methods such as Euler and Runge-Kota has shown a stable mathematical and reliable behavior for regular and uncomplicated problems.

- It has been concluded that appropriate method should be chosen based on the nature of the equation, the complexity level, available dataset and the computational resources of the machine. It has also been found that the training time for AI models is quite more compared to traditional methods but predicting speed is much faster than traditional models after completion of training for the intelligent methods***(Mishra.et al,2022)***..

* In term of performance, combination of the AI and traditional numerical method hybrid models was observed to be the most accurate and efficient approach.

* The results indicate that to assess intelligent methods accuracy with the classification problem appropriate evaluation metrics such as Accuracy, Recall and F1-Score were needed to be implemented, as for regression and numerical approximation problems MSE and RMSE should be implemented.

* The study demonstrates that the continuous progress in the field of AI can create a breakthrough in solving the differential equations and their use in science and engineering disciplines**(*Ren*.etal,2025)**..

## 6.Future Work

More efficient and accurate hybrid frameworks that combine traditional numerical approaches with artificial intelligence algorithms for solving differential equations need to be explored in this area. Additional research is expected to concentrate on how to accelerate training speed, ensure training stability and enhance the generalization of ANNs and PINNs, particularly in highly nonlinear and high dimensional systems. Adaptive optimization algorithms, automatic hyperparameter tuning and parallel computation methods may also be researched and developed further to cut down on the high computational cost associated with such problems. Other promising directions include applying these intelligent models to engineering, medical, climate and economic problems and many other scientific fields with complex differential systems frequently occurring. Comparing the results with traditional numerical methods and other machine learning algorithms under standard datasets will provide an objective evaluation of the performance of the AI-based solvers.

**Compliance with ethical standards**

*Disclosure of conflict of interest*

The authors declare that they have no conflict of interest.

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